Revision answers: Algebra, functions and equations (Topics 1 & 2)

Coursebook chapters: 1–7

1. $2x + 1 = 3^2 = 9 \quad \therefore \; x = 4$ *[4 marks]*

2. $\left(\frac{x}{3}\right)^6 + 6\left(\frac{x}{3}\right)^5(-2) + 15\left(\frac{x}{3}\right)^4(-2)^2 = \frac{x^6}{729} - \frac{4x^5}{81} + \frac{20x^4}{27}$ *[5 marks]*

3. (a) $\mathbb{R}$

(b) $f^{-1}(x) = \frac{1}{3}e^{2x}$

(c) $f^{-1}(5) = \frac{1}{3}e^{10}$ *[5 marks]*

4. $8a - b = -13, \frac{a}{8} + b = \frac{9}{4} \quad \therefore \; a = -\frac{86}{65}, b = \frac{157}{65}$ *[5 marks]*

5. (a) $\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 5 = \left(x - \frac{3}{2}\right)^2 + \frac{11}{4}$

(b) $x^2 - 3x + \geq \frac{11}{4}, \quad \therefore \; 0 < f(x) \leq \frac{4}{11}$ *[5 marks]*

6. The discriminant $= 16 - 4k(k + 1) = 0$, so $k^2 + k - 4 = 0 \quad \therefore \; k = \frac{-1 \pm \sqrt{17}}{2}$ *[5 marks]*

7. (a) Arithmetic series, $u_1 = 500$, $d = 25$, $S_{20} = 14750$

(b) Geometric, $u_1 = 500$, $r = 1.05$, $u_n = 500 \times 1.05^{n-1} > 1000 \quad \Rightarrow \quad n = 16$ (from GDC)

(c) $\frac{500\left(1.05^n - 1\right)}{0.05} > \frac{n}{2}(1000 + 25(n - 1)) = 5000$ gives $n = 27$ days *[9 marks]*

8. (a) $g(x) > 1$

(b) $3(4e^x + 1)^2 = 75 \Rightarrow 4e^x + 1 = 5 \Rightarrow x = 0$ *[8 marks]*

9. (a) $p(3) = 0$

(b) $p(x) = (x - 3)(2x^2 + x - 3) = (x - 3)(2x + 3)(x - 1)$

(c) Cubic graph with x-intercepts -1.5, 1, 3 and y-intercept 9. *[9 marks]*

10. (a) Reflection in the y-axis and vertical stretch with scale factor 3; zeros (2, 0), (0, 0), (−4, 0); turning points (1, −6), (−2, 18).

(b) Horizontal stretch with scale factor $\frac{1}{2}$ and a vertical translation by 1 unit; y-intercept (0, 1); turning points $\left(-\frac{1}{2}, -1\right)$, (1, 7). *[6 marks]*

11. $2^x - \frac{4}{2^x} = 3 \Rightarrow 2^{2x} - 4 = 3 \times 2^x \Rightarrow 2^{2x} - 3 \times 2^x - 4 = 0 \Rightarrow (2^x - 4)(2^x + 1) = 0$

$\therefore 2^x = 4, x = 2$ *[5 marks]*

12. (a) $x = \frac{b}{2}$

(b) $f(x) = f^{-1}$ is equivalent to $f(x) = x$ (because the two graphs cross on the line $y = x$.

$\frac{x+3}{2x-5} = x \Rightarrow x = -0.436$ or 3.44 *[5 marks]*