Revision answers: Geometry (Topics 3 & 4)

Coursebook chapters: 8–11

1. $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta = \frac{1}{2}(5^2)(1.2 - \sin 1.2) = 3.35 \text{ cm}^2$ *[4 marks]*

2. $\tan(2\theta) = \sqrt{3}$, $0 \le 2\theta \le 360°$

$\arctan\sqrt{3} = 60°$

$\Rightarrow \quad 2\theta = 60°$ or $60 + 180 = 240°$

$\Rightarrow \quad \theta = 30°, 120°$ *[5 marks]*

3. $\overrightarrow{AB} = \begin{pmatrix} -3 \\ -5 \\ 0 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} -4 \\ -7 \\ 1 \end{pmatrix}$

$$\cos\hat{A} = \frac{\begin{pmatrix} -3 \\ -5 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -7 \\ 1 \end{pmatrix}}{\sqrt{3^2 + 5^2 + 0^2}\sqrt{4^2 + 7^2 + 1^2}} = 0.992$$

$$\therefore \quad \hat{A} = 7°$$

$\overrightarrow{BA} = \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$

$$\cos\hat{B} = \frac{\begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}}{\sqrt{3^2 + 5^2 + 0^2}\sqrt{1^2 + 2^2 + 1^2}} = -0.910$$

$$\therefore \quad \hat{B} = 156°$$

$$\hat{C} = 180 - 7 - 156 = 17°$$ *[9 marks]*

4. $\frac{\sin 29}{6.5} = \frac{\sin \hat{A}}{12}$

$\sin \hat{A} = 0.895$

$\therefore \ \hat{A} = 63.5°$ or $180 - 63.5 = 116.5°$ *[5 marks]*

5. amplitude $= \frac{5-(-2)}{2} = 3.5 \ \ \therefore \ A = 3.5$

$B = \frac{5+(-2)}{2} = 1.5$

half-period $= \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3} \ \ \therefore k = \frac{\pi}{\frac{\pi}{3}} = 3$ *[5 marks]*

6. $2x + 30 \in [30, 750]$; $2x + 30 = 60, 300, 420, 660 \ \ \therefore \ x = 15, 135, 195, 315$ *[5 marks]*

7. (a) $(1 - 2\sin^2 x) - 3\sin x + 1 = 0 \ \Leftrightarrow \ 2\sin^2 x + 3\sin x - 2 = 0$

(b) $\sin x = \frac{1}{2} \ \ \therefore \ x = \frac{\pi}{6}, \frac{5\pi}{6}$ *[7 marks]*

8. Cosine rule: $(2x + 1)^2 = (x + 1)^2 + (x + 3)^2 - 2(x + 1)(x + 3)\left(-\frac{1}{2}\right)$

$\Rightarrow \ x^2 - 8x - 12 = 0$

$\therefore \ x = 9.29$ *[6 marks]*

9. (a) Write BC = $5x$, AC = $4x$, then:

$\frac{\sin 2\theta}{5x} = \frac{\sin \theta}{4x} \ \Rightarrow \ \frac{2\sin\theta\cos\theta}{5} = \frac{\sin\theta}{4} \ \Rightarrow \ 8\cos\theta = 5$ (as $\sin\theta \neq 0$)

$\therefore \ \theta = 51.3°$

(b) 51.3°, 103°, 26.0° *[6 marks]*

10. (a) $\boldsymbol{r} = t\begin{pmatrix} 4 \\ -1 \\ -2 \end{pmatrix}$

(b) $(8, -2, -4)$

(c) $\cos\theta = \dfrac{8-5-6}{\sqrt{4+25+9}\sqrt{16+1+4}} = -0.106 \quad \therefore \theta = 96.1°$

(d) $\text{AB} = \sqrt{4^2+1^2+2^2} = \sqrt{21}$, distance $= \sqrt{21}\sin(180 - 96.1) = 4.56$ *[12 marks]*

11. (a) $\begin{pmatrix} \cos\theta \\ \sin\theta \\ -\sin\theta \end{pmatrix} \cdot \begin{pmatrix} \cos\theta \\ -\sin\theta \\ \cos\theta \end{pmatrix} = 0$

$\Leftrightarrow \quad \cos^2\theta - \sin^2\theta - \sin\theta\cos\theta = 0$

$\Leftrightarrow \quad \cos 2\theta - \dfrac{1}{2}\sin 2\theta =$

$\Leftrightarrow \quad 2\cos 2\theta = \sin 2\theta$

$\Leftrightarrow \quad \tan 2\theta = 2$

(b) $\dfrac{2\tan\theta}{1-\tan^2\theta} = 2$

$\Leftrightarrow \quad 2\tan^2\theta + 2\tan\theta - 1 = 0$

$\Leftrightarrow \quad \tan\theta \ \dfrac{2 \pm \sqrt{12}}{4} = \dfrac{-1 \pm \sqrt{3}}{2}$

$\theta \in \left(0, \dfrac{\pi}{2}\right) \quad \Rightarrow \quad \tan\theta > 0$

$\therefore \quad \tan\theta = \dfrac{-1+\sqrt{3}}{2}$ *[9 marks]*