# Chapter notes: 17 Probability

## Overview

*As the 'From another perspective' box on page 507 highlights, probability is one of the most recent mathematical disciplines. In many countries it is not taught until the final year at school, so you may have students who have not encountered it before, as well as students who will be familiar with most of the sections A to E. If students are not familiar with these concepts, it will require around 12 hours of teaching time.*

## Introductory problem

The introductory problem highlights the importance of understanding the sample space and conditional probabilities, as well as using precise language. The 'Theory of knowledge issues' point on page 543 invites students to think about the role of language in mathematics. The 'Research explorer' box on the same page refers to a game show involving cars and goats; there are many online simulations of the game. The worked solution is given at the end of the chapter, page 543; the idea being that students should be able to answer the question using the methods covered in the chapter.

## 17A Empirical probability, p507

In question 4, the overall percentage will only be 50% if the samples are of equal size.

## 17B Theoretical probability, p510

The 'Research explorer' box on page 510 mentions the 'law of large numbers'. This states that the average of the results obtained from repeating an experiment a large number of times, nearly always tends to the theoretical probability. Students should be aware that, for example, repeatedly rolling a 'six' on a die is possible, although the probability tends to zero.

The 'Research explorer' box on page 512 refers to the Schrödinger's Cat. In this thought experiment we cannot tell whether a cat in a box is dead or alive, so we could say that it is alive with certain probability.

An example for question 4 might be $A$ = *rolling a prime number on a standard die* and $B$ = *rolling an even number on a standard die*.

## 17C Combined events and Venn diagrams, p515

The basic ideas about Venn diagrams are covered in Prior learning section J. This section mainly looks at the formula connecting the intersection and union of two sets:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The 'Theory of knowledge issues' box brings up again the point about the use of language in mathematics.

## 17D Tree diagrams and finding intersections, p520

We use tree diagrams to introduce the concept of conditional probability; this is probability that depends on some previous information (such as burgers being one of the options for lunch).

*Hints for grade 7 questions:*

**7.** Denote the number of black disks by $b$ and draw a tree diagram.

## 17E Independent events, p525

We define independent events as those having $P(A|B) = P(A)$. This corresponds to the intuitive notion of independence. The formula $P(A \cap B) = P(A)P(B)$ can then be derived using the definition of conditional probability.

Many textbooks quote the latter formula as the definition of independent events. The 'Theory of knowledge issues' box asks students to consider the difference between a definition and a property. The distinction is not always clear; we can have several equivalent definitions. The answer to the last question in the 'Theory of knowledge issues' box is 'No': there are shapes other than circles that have constant width, one example being the British 50 pence coin.

## 17F Conditional probability, p528

This section continues the discussion of conditional probability from section 17D.

*Hints for grade 7 questions:*

**10.** Label $P(A \cap B) = x$ and draw a Venn diagram.

## 17G Further Venn diagrams, p533

This section extends the work on Venn diagrams to include more than two sets.

The 'Theory of knowledge issues' box invites students to think about relative merits of using formulae and diagrammatic representations. Many people worry that visual arguments are not valid in mathematics, but this is not necessarily the case.

## 17H Selections with and without replacement, p538

This section, mentioned explicitly in the Standard Level syllabus, highlights the situation when the probabilities can stay constant, even though the events are not independent.

*Hints for the grade 7 questions:*

**6.** (b) Consider the probability of getting all balls the same colour.