Chapter notes: 12 Basic differentiation and its principles

Overview

This chapter is fundamental to all of the other calculus chapters. It begins by developing a graphical interpretation of derivatives, then it builds up a reasonable range of functions which can be differentiated. Finally, these are applied to the common problems of optimisation and finding tangents. We think approximately 12 hours of teaching time is required.

Introductory problem

This problem is designed to result in the need for optimisation. You might like to use it as an opportunity to get students to think about other situations where optimisation is desired. The worked solution is given at the end of the chapter, page 389; the idea being that students should be able to answer the question using the methods covered in the chapter.

12A Sketching derivatives, p348

This section introduces derivatives graphically. As well as developing an understanding of the topic, several examination questions have tested this skill. Some interesting examples to use in question 3 might be:

	Situation when true	**Situation when false**
(a)	$y = x^2, x > 0$	$y = x^3, x < 0$
(b)	$y = \frac{1}{x}, x < 0$	$y = x^2, x < 0$
(d)	$y = x^2, x = 0$	$y = x^3, x = 0$
(e)	$y = e^x$	$y = -e^{-x}$
(f)	$y = x^2$	$y = \lvert x \rvert$

12B Differentiation from first principles, p356

Hints for the grade 7 questions:

4. Use $g(x) = kf(x)$ in the formula for differentiation from first principles.

12C-D

There are no specific teacher notes for these sections.

12E Differentiating trigonometric functions, p368

If x is measured in degrees then, $\frac{d}{dx}(\sin x) = \frac{\pi}{180}\cos x$. It might be useful to get the stronger students to prove this.

12F Differentiating exponential and natural logarithm functions, p370

One definition of the value e is given by Key point 12.9. Another is given in section 2C. A third comes from the series definition that $e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}$ ... . You might like to use this as an opportunity to discuss how mathematicians have some freedom to choose what their fundamental definitions are.

Question 7 provides good revision of the rules of logarithms and indices.

Hints for the grade 7 questions:

6. After differentiating, form a quadratic equation.

12G Tangents and normals, p372

The $y - y_1 = m(x - x_1)$ version of the equation for a straight line (covered in Prior learning section R) will be extremely useful in this section.

Hints for grade 7 questions:

6. Be careful to not confuse the x on the curve with the x on the normal.

7. Find the equation of the tangent at $x = a$, then find the coordinates of P and Q.

8. Find the equation of the tangent and solve simultaneously with the original curve. Remember that you do not need to find all solutions to the resulting cubic.

12H Stationary points, p376

When asked to justify the nature of stationary points, it is fine to use the second derivative or to find the sign of the derivative at points close to the stationary point.

Hints for grade 7 questions:

8. Use the $\frac{d^2y}{dx^2}$ method for determining the stationary point.

12I General points of inflexion, p382

Hints for grade 7 questions:

5. First solve $\frac{d^2y}{dx^2} = 0$.

6. $f''(x) = 0$ corresponds to a stationary point on the graph of $f'(x)$.

12J Optimisation, p384

Students frequently forget to check end points or poles for the global maximum and minimum.

Hints for grade 7 questions:

9. (b) This is asking you to maximise $\frac{dV}{dt}$, so solve $\frac{d^2V}{dt^2} = 0$.

11. (c) This is asking you to maximise $\frac{\mathrm{d}V}{\mathrm{d}t}$.

12. (c) Solve a cubic inequality graphically.

(d) Form a new function as the difference between energy production and energy usage.

