Self-assessment answers: 13 Basic integration and its applications

1. $\int \frac{\sqrt{x}-3x^2}{4x^3}\,\mathrm{d}x = \frac{1}{4}\int\left(\frac{\sqrt{x}}{x^3}-\frac{3x^2}{x^3}\right)\mathrm{d}x$

$= \frac{1}{4}\int\left(x^{-\frac{5}{2}}-3x^{-1}\right)\mathrm{d}x$

$= -\frac{1}{6}x^{-\frac{3}{2}}-\frac{3}{4}\ln x + c$ *[6 marks]*

2. $\int_0^{\pi/3} 4\cos x\,\mathrm{d}x = [4\sin x]_0^{\pi/3} = 2\sqrt{3}$ *[5 marks]*

3. $\int_0^a \mathrm{e}^x - x\,\mathrm{d}x = 6.8$

$\Rightarrow \left[\mathrm{e}^x - \frac{x^2}{2}\right]_0^a = 6.8$

$\Rightarrow \mathrm{e}^a - \frac{a^2}{2} - 1 = 6.8$

$\Rightarrow a = 2.36$ (GDC) *[6 marks]*

4. (a) $f(x) = x^2 - 6x + c$

Passes through (2, 0) $\Rightarrow$ $0 = 4 - 12 + c$ $\Rightarrow$ $c = 8$

$f(x) = x^2 - 6x + 8$

(b) At (2, 0), $f'(x) = -2$

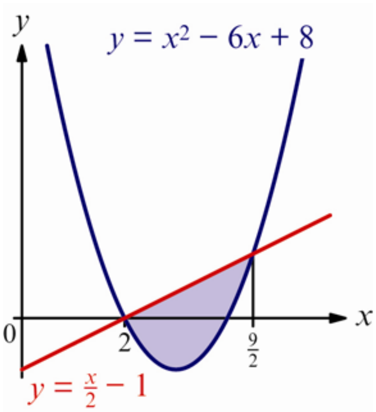
So normal has gradient $\frac{1}{2}$ and therefore has equation $y = \frac{x}{2} - 1$.

(c) Normal meets the curve when $\frac{x}{2} - 1 = x^2 - 6x + 8$.

$$\Rightarrow \; x^2 - \frac{13}{2}x + 9 = 0$$

$$\Rightarrow \; (x-2)\left(x - \frac{9}{2}\right) = 0$$

Intersections are (2, 0) and $\left(\frac{9}{2}, \frac{5}{4}\right)$.



Area enclosed is given by $\left|\int_2^{9/2} x^2 - \frac{13}{2}x + 9 \, \mathrm{d}x\right|$

$$= \left|\left[\frac{x^3}{3} - \frac{13}{4}x^2 + 9x\right]_2^{9/2}\right|$$

$$= \left|\frac{243}{8} - \frac{1053}{16} + \frac{81}{2} - \left(\frac{8}{3} - 13 + 18\right)\right|$$

$$= \frac{125}{48} = 2.6$$

[13 marks]