Self-assessment answers: 11 Vectors

1. (a) $\overrightarrow{MP} = \overrightarrow{MB} + \overrightarrow{BP} = \frac{1}{3}\boldsymbol{a} + \frac{3}{4}\boldsymbol{b}$

$\overrightarrow{QN} = \overrightarrow{QD} + \overrightarrow{DN} = \frac{1}{2}\boldsymbol{b} + \frac{2}{9}\boldsymbol{a}$

(b) $\overrightarrow{MP} = \frac{1}{12}(4\boldsymbol{a} + 9\boldsymbol{b}), \overrightarrow{QN} = \frac{1}{18}(4\boldsymbol{a} + 9\boldsymbol{b})$

$\therefore\ \overrightarrow{MP} = \frac{3}{2}\overrightarrow{QN}$ and so they are parallel. *[6 marks]*

2. (a) $\overrightarrow{BC} = \begin{pmatrix} -10 \\ -7 \\ -1 \end{pmatrix}$ so $BC = \sqrt{150} = 5\sqrt{6}$

(b) $\overrightarrow{CB} = \begin{pmatrix} 10 \\ 7 \\ 1 \end{pmatrix}, \overrightarrow{CA} = \begin{pmatrix} 4 \\ 6 \\ -1 \end{pmatrix}$

$A\hat{C}B = \arccos\left(\frac{\overrightarrow{CB} \cdot \overrightarrow{CA}}{CB \times CA}\right)$

$= \arccos\left(\frac{81}{5\sqrt{6} \times \sqrt{53}}\right)$

= 0.431 radians (24.7°) *[7 marks]*

3. (a) Direction vector is $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, so the vector equation of l_1 is $\boldsymbol{r} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ (or equivalent).

In Cartesian form, this is $x - 4 = -y = \frac{3 - z}{2}$.

(b) Intersection where $\begin{pmatrix}4\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\-2\end{pmatrix}=\begin{pmatrix}-4\\5\\-5\end{pmatrix}+t\begin{pmatrix}2\\-1\\2\end{pmatrix}$

$$\Rightarrow \begin{cases} 4+\lambda=-4+2t & (1)\\ -\lambda=5-t & (2)\\ 3-2\lambda=-5+2t & (3)\end{cases}$$

(1) + (2) $\Rightarrow$ $4=1+t$

$\Rightarrow$ $t=3$

(2) $\Rightarrow$ $\lambda=-2$

(3) $\Rightarrow$ $3+4=-5+6$: False

These are skew lines and do not intersect. *[7 marks]*

4. (a) $\boldsymbol{r}_A=(3\boldsymbol{i}+4.5\boldsymbol{j})t$ km *[1 mark]*

(b) $\boldsymbol{r}_B=16\boldsymbol{i}+23\boldsymbol{j}+(-5\boldsymbol{i}-\boldsymbol{j})t$ km

Then, $\overrightarrow{AB}=\boldsymbol{r}_B-\boldsymbol{r}_A$

$=16\boldsymbol{i}+23\boldsymbol{j}+(-5\boldsymbol{i}-\boldsymbol{j})t-(3\boldsymbol{i}+4.5\boldsymbol{j})t$

$=(16-8t)\boldsymbol{i}+(23-5.5t)\boldsymbol{j}$ *[3 marks]*

(c) Distance between cars = $|\overrightarrow{AB}|$

$$=\sqrt{(16-8\times2.5)^2+(23-5.5\times2.5)^2}$$

$$=\frac{5\sqrt{65}}{4}\approx 10.1 \text{ km}$$ *[2 marks]*

(d) For cars to meet, there must exist a t for which $|\overrightarrow{AB}|=0$, or equivalently for which $|\overrightarrow{AB}|^2=0$.

$(16-8t)^2+(23-5.5t)^2=0$

$\Leftrightarrow$ $256-256t+64t^2+529-523t+\frac{121}{4}t^2=0$

$\Leftrightarrow$ $377t^2-2036t+3140=0$

The discriminant $D=b^2-4ac<0$ and therefore no solutions exist. *[4 marks]*