Self-assessment answers: 7 Binomial expansion

If there is a question you can't do, this table shows you which section in the textbook can help you.

Question	Section or Worked example
1. (a), (b)	Section 8B (Worked example 8.2)
1. (c)	Worked example 8.4
2. (a)	Section 8B
2. (b)	Section 8D
3.	Section 8B
4. (b)	Section 8B
4. (c)	Section 8B (Worked example 8.2)

1. (a) $\binom{7}{4}\times(-2)^4 = 560$

(b) $\binom{10}{7}(2)^3(5)^7 = 75\ 000\ 000$

(c) $x^{13-k}\left(\frac{1}{x}\right)^k = x^3$

$\Rightarrow\ 13 - 2k = 3$

$\Rightarrow\ k = 5$

The coefficient is $\binom{13}{5}(1)^5(-1)^8 = 1287$. *[4 marks]*

2. (a) $2^5 - \binom{5}{1}\times 2^4x + \binom{5}{2}\times 2^3x^2 = 32 - 5\times 16x + \frac{5\times 4}{2}\times 8x^2$

$= 32 - 80x + 80x^2$

(b) $2 - x = 1.99$ when $x = 0.01$.

$32 - 80\times 0.01 + 80\times 0.01^2 = 32 - 0.8 + 0.008$

$= 31.208$ *[7 marks]*

3. (a) $(1 + 4x + 6x^2 + 4x^3 + x^4) + (1 - 4x + 6x^2 - 4x^3 + x^4)$

$= 2 + 12x^2 + 2x^4$

(b) Let $x = \sqrt{2}$:

$(\sqrt{2} + 1)^4 + (\sqrt{2} - 1)^4 = 2 + 12(\sqrt{2})^2 + 2(\sqrt{2})^4$

$= 2 + 12(2) + 2(4) = 34$ *[7 marks]*

4. (a) $x = 3 - \dfrac{1}{x} \quad \therefore \quad x + \dfrac{1}{x} = 3$

(b) (i) $\left(x + \dfrac{1}{x}\right)^2 = x^2 + 2x\left(\dfrac{1}{x}\right) + \left(\dfrac{1}{x}\right)^2$

$= x^2 + 2 + \dfrac{1}{x^2}$

$\left(x + \dfrac{1}{x}\right)^3 = x^3 + 3x^2\left(\dfrac{1}{x}\right) + 3x\left(\dfrac{1}{x}\right)^2 + \left(\dfrac{1}{x}\right)^3$

$= x^3 + 3x + \dfrac{3}{x} + \dfrac{1}{x^3}$

(ii) $\left(x + \dfrac{1}{x}\right)^2 = 9 \Rightarrow 9 = x^2 + 2 + \dfrac{1}{x^2}$

$\Rightarrow x^2 + \dfrac{1}{x^2} = 7$

$\left(x + \dfrac{1}{x}\right)^3 = 27 \Rightarrow 27 = x^3 + 3x + \dfrac{3}{x} + \dfrac{1}{x^3}$

$\Rightarrow x^3 + \dfrac{1}{x^3} = 27 - 3\left(x + \dfrac{1}{x}\right)$

$= 27 - 3(3) = 18$

(c) The constant term appears when the powers of x and $\frac{1}{x}$ are equal:

$$\binom{n}{k} x^k \left(\frac{1}{x}\right)^{n-k} = \binom{n}{k} x^{2k-n}$$

$$\Rightarrow \quad 2k - n = 0$$

$$\Rightarrow \quad k = \frac{n}{2}$$

So $\binom{n}{n/2} = 70.$

Using table from GDC:

n	$\binom{n}{n/2}$
2	2
4	6
6	20
8	70

$\therefore \; n = 8$ *[12 marks]*

