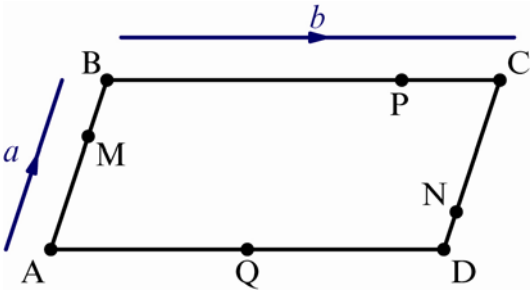
Self-assessment: 11 Vectors

1. In the diagram below, $\overrightarrow{AB} = \overrightarrow{DC} = \boldsymbol{a}$ and $\overrightarrow{BC} = \overrightarrow{AD} = \boldsymbol{b}$. Q is the midpoint of AD and points M, N, P and Q are such that AM : MB = 2 : 1, DN : NC = 2 : 7, BP : PC = 3 : 1.



(a) Express $\overrightarrow{MP}$ and $\overrightarrow{QN}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(b) Hence show that (MP) and (QN) are parallel.

(accessible to students on the path to grade 3 or 4) [6 marks]

2. A triangle has vertices with coordinates A(3, 6, 1), B(9, 7, 3) and C(−1, 0, 2).

(a) Find the length of the side BC.

(b) Calculate the size of the angle $A\hat{C}B$.

(accessible to students on the path to grade 3 or 4) [7 marks]

3. Line l_1 passes through points with coordinates (4, 0, 3) and (5, −1, 1). Line l_2 has equation

$$\boldsymbol{r} = \begin{pmatrix} -4 \\ 5 \\ -5 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

(a) Find the equation of l_1.

(b) Determine whether l_1 and l_2 intersect, and if so, at what point.

(accessible to students on the path to grade 5 or 6) [7 marks]

4. Car *A* starts from the origin and moves with velocity $\boldsymbol{v_A} = (3\boldsymbol{i} + 4.5\boldsymbol{j})$ kmh^{-1}.

(a) Write down the position vector of car *A* after *t* hours.

Car *B* starts from the position $(16\boldsymbol{i} + 23\boldsymbol{j})$ km and moves with velocity $\boldsymbol{v_B} = (-5\boldsymbol{i} - \boldsymbol{j})$ kmh^{-1}.

(b) Find an expression for $\overrightarrow{AB}$.

(accessible to students on the path to grade 3 or 4)

(c) Find the distance between the cars after 2.5 hours.

(accessible to students on the path to grade 5 or 6)

(d) Show that the cars never meet.

(accessible to students on the path to grade 7)

[10 marks]