Chapter notes: 11 Vectors

Overview

Introductory problem

The introductory problem highlights some of the difficulties posed by three-dimensional problems. It can be solved using Pythagoras and the cosine rule. But, before applying these rules we need to ask whether the two diagonals cross at all. Vectors will provide a way to answer this question without having to rely on our spatial intuition. The worked solution is given at the end of the chapter, page 343; the idea being that students should be able to answer the question using the methods covered in the chapter.

11A Positions and displacements, p295

This section introduces the concept of vectors as 'arrows', the graphical representation of vector operations, and the representation of vectors using components. The distinction is made between 'vector displacements' and 'position vectors'; this distinction is important for understanding problems involving equations of lines in section F. Worked example 11.4 and question 10 show how vectors can be used to describe geometrical properties; this application of vectors is emphasised in the new syllabus.

11B Vector algebra, p304

In this section we look at vectors in a more abstract way, emphasise that the same algebraic rules apply no matter what the vectors represent. We also look at the connection between algebraic and diagrammatic representations, which is really important for dealing with lines and planes problems in the next chapter.

11C Distances, p309

In this section we return to vectors describing positions and displacements. The diagonal of a cube links back to Key point 10.7 on p272; you can use this to discuss different approaches to solving three-dimensional problems.

Unit vectors are not used very much in this course. However, the ability to find a vector of a given length in a given direction, as in Worked example 11.7 (b), is useful.

Hints for the grade 7 questions:

11. Write $\overrightarrow{\mathrm{AB}}$ in terms of t. You may want to ask students to think about all possible positions of point B (they form a straight line).

12. Write $\overrightarrow{\mathrm{PQ}}$ in terms of t. Note that you can minimise the square of the distance rather than the distance itself. If you discussed the straight line interpretation in question 11, then this question can be interpreted as finding the shortest distance from a point to a line. We will meet this again in Worked example 11.19.

11D Angles, p313

We have introduced the scalar product as the quantity $a_1b_1 + a_2b_2 + a_3b_3$, which comes up when we try to calculate an angle between vectors (see Key points 11.6 and 11.7). The other way of defining it is as $|\boldsymbol{a}||\boldsymbol{b}| \cos \theta$ and then showing how to calculate it using components. In the next section we will mention the second definition and look at properties of the scalar product as an algebraic operation.

The derivation of the formula in Key point 11.6 is not too difficult – it may be worth going through. (See Fill-in proof sheet 8, which you could make even simpler by looking at the two-dimensional case. The questions and the 'Theory of knowledge issues' box at the end of the proof sheet explore the idea of definition in mathematics.)

11E Properties of the scalar product, p318

This section looks at the scalar product as an algebraic operation and compares its properties to 'normal' multiplication. The special case of perpendicular vectors will be used widely in the next chapter.

The 'Theory of knowledge issues' box brings up the question of fourth dimension. Some students may have encountered the idea of the fourth dimension being time (which comes from Einstein's Theory of Relativity). But they could also be encouraged to think of dimension simply as the number of coordinates required to define a position of a point.

Hints for grade 7 questions:

11. (b) You need to use the result $\boldsymbol{a} \cdot \boldsymbol{a} = |\boldsymbol{a}|^2$.

12. (a) You don't need to find the equation of the line AB, it is sufficient to show that $\overrightarrow{\mathrm{OB}}$ is parallel to $\overrightarrow{\mathrm{OA}}$.

(b) Express $\overrightarrow{\mathrm{BC}}$ and $\overrightarrow{\mathrm{BA}}$ in terms of λ.

11F Vector equation of a line, p322

This section encourages thinking about the vector equation of a line as a way of describing all possible points which lie on the line. This may be different from the way students were taught about the equation of the line before: $y = mx + c$ is a way to calculate the y-coordinate given the x-coordinate. Students often expect the equation of the line in three dimensions to be of the form $z = ax + by + c$. You can point out later that this type of equation defines a plane. You may want to discuss the idea of dimension and degrees of freedom. If we know the x-coordinate of a point on the line, this determines the other two coordinates (one degree of freedom – and this is the case in two and three dimensions). But to define a point in the plane we need to know two coordinates.

Understanding how to use an expression for a general point on the line, as in Worked example 11.15, is essential for solving harder problems.

11G Solving problems involving lines, p330

In this section we look at the most common problems involving equations of lines. The problem in Worked example 11.19 (b) – finding the shortest distance from a point to a line – can also be attempted by writing a general expression for the distance (as in Worked example 11.15) and minimising the resulting quadratic expression.

For students already familiar with using vector displacements and velocities, you could use this to introduce the idea of the equation of a line (it is a description of the path of an object moving with constant velocity). Solving problems, such as the one in Worked example 11.20, is clearly mentioned in the SL syllabus.

Hints for grade 7 questions:

5. Follow the method in Worked example 11.19. This question is red because the method is not broken down into steps.

7. This requires the same method as for question 5.

8. Draw a diagram and add information that you find as you work through.

9. (c) Draw a diagram and notice that $\overrightarrow{\mathrm{QM}} = \overrightarrow{\mathrm{MR}}$.

10. Notice that the coordinates of P are obvious. You will need to use a general expression for $\left|\overrightarrow{\mathrm{PR}}\right|$.

